TOPOI, LOGIC & FORCING

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We will follow the book *Sheaves in Geometry and Logic* by Saunders Mac Lane and Ieke Moerdijk.

1. Introduction, 26.04.22
Kim Nguyen

Give your favorite definition of elementary topos. Indicate how to do logic in a topos and give first examples.

2. First properties of topoi, 03.05.22
Kristina Dengler

Discuss the factorization of a morphism onto its image in a topos, iv.6, and internal internal Heyting algebra objects in a topos, iv.8. Prove the existence of the epi-mono factorization of a morphism, iv.6.1 and the Heyting algebra structure on subobjects, iv.8.1 (both internal and external).

3. Lawvere-Tierney topologies, 10.05.22
Katharina Schneider

Motivate and define Lawvere-Tierney topologies and sheaves associated to a Lawvere-Tierney topology on a topos, v.1 and v.2. Prove Theorem v.3.1.

4. Sets, 17.05.22
Alexandra Pröls

Explain natural numbers objects in a topos. Recall the notion of a Boolean topos and prove Proposition vi.1.1. Show that the double negation operator defines a Lawvere-Tierney topology, Theorem vi.1.3. Discuss filter quotients v.9.

5. The Cohen topos, 24.05.22
Pier Federico Pacchiarotti

Show that the filter-quotient topos of a Boolean topos is two-valued, vi.1.6. Explain the poset of forcing conditions and prove Theorem vi.2.1.

6. Cardinal inequalities, 31.05.22
Jonas Linssen

Explain the object of epimorphisms and prove Corollary vi.3.3. Introduce the Souslin property and prove Proposition vi.3.6. Complete the argument that the Cohen topos is a model of set theory in which the Continuum Hypothesis fails.

7. The axiom of choice, 14.06.22
Benedikt Fröhlich

Explain the axiom of choice in a topos, vi.1 towards the end. Define the notion of well-pointedness and prove Propositions vi.1.7 an vi.1.8 as well as Corollary vi.1.9. Prove Theorem vi.4.1.

8. The Mitchell-Bénabou language, 21.06.22
Areeb Shah Mohammed

This talk will explain languages. Follow vi.5 for the description of the Mitchell-Bénabou language of a topos. Choose any source you like to formally introduce languages. Also explain semantics as in the beginning of vi.6.

9. Kripke-Joyal semantics, 28.06.22
Roland Herrmann

Prove Theorem vi.6.1. Explain the connection of the rules of Kripke-Joyal semantics to the notion of forcing developed so far. Translate categorical notions of topoi into formulas as described after Theorem vi.6.1, making the connection to the previous talk.

10. Sheaf semantics, 05.07.22
Angela Pellone

Formally introduce Grothendieck topoi iii.1 and iii.2. If time permits construct the sheafification functor. Explain Kripke-Joyal semantics in a Grothendieck topos and prove Theorem vi.7.1.

11. Real numbers in a topos, 12.07.22
Heiko Braun

Explain how to define Dedekind reals in a topos and prove Theorem vi.8.2.

12. Brouwer's Theorem, 19.07.22

Prove Brouwer's Theorem that all functions from the reals to themselves are continuous, vi.9.3. Make precise why this holds in intuitionistic logic.